

**LEFT-RIGHT GAUGE SYMMETRY  
AT THE TEV ENERGY SCALE\***

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Two first examples beyond the standard model are given which exhibit left-right symmetry ( $g_L = g_R$ ) and supersymmetry at a few TeV, together with gauge-coupling unification at around  $10^{16}$  GeV.

**1. Introduction**

What lies beyond the standard model at or below the TeV energy scale? One very well-motivated possibility is supersymmetry. In particular, the minimal supersymmetric standard model (MSSM) is being studied by very many people. Another possibility is left-right gauge symmetry, but there are a lot fewer advocates here and for good reason, as I will explain in this talk. I will also discuss how these problems may be overcome, assuming both supersymmetry and left-right gauge symmetry at the TeV energy scale.

There are two problems with the conventional left-right gauge model at the TeV energy scale with or without supersymmetry. One is the unavoidable occurrence of flavor-changing neutral currents (FCNC) at tree level. The other is the lack of gauge-coupling unification which is known to be well satisfied by the MSSM.<sup>1</sup> In this talk, I will offer two new models.<sup>2,3</sup> Both allow the gauge couplings to be unified at around  $10^{16}$  GeV. The second has the added virtue of being free of FCNC at tree level. Hence left-right gauge symmetry at a few TeV should be considered a much more attractive possibility than was previously recognized.

**2. Origin of FCNC in Left-Right Models**

Consider the gauge symmetry  $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  which breaks down to the standard  $SU(3)_C \times SU(2)_L \times U(1)_Y$  at  $M_R \sim \text{few TeV}$  with particle

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content given by

$$Q \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L \sim (3, 2, 1, 1/6), \quad Q^c \equiv \begin{pmatrix} d^c \\ u^c \end{pmatrix}_L \sim (\bar{3}, 1, 2, -1/6), \quad (1)$$

$$L \equiv \begin{pmatrix} \nu \\ l \end{pmatrix}_L \sim (1, 2, 1, -1/2), \quad L^c \equiv \begin{pmatrix} l^c \\ \nu^c \end{pmatrix}_L \sim (1, 1, 2, 1/2). \quad (2)$$

Note that each generation of quarks and leptons (*i.e.*  $Q + Q^c + L + L^c$ ) fits naturally into a **16** representation of  $\text{SO}(10)$ . In order for the quarks and leptons to obtain nonzero masses, a scalar bidoublet

$$\eta \equiv \begin{bmatrix} \eta_1^0 & \eta_2^+ \\ \eta_1^- & \eta_2^0 \end{bmatrix} \sim (1, 2, 2, 0) \quad (3)$$

is required. Consider the interaction of  $\eta$  with the quarks:

$$QQ^c\eta = dd^c\eta_1^0 - ud^c\eta_1^- + uu^c\eta_2^0 - du^c\eta_2^+. \quad (4)$$

If there is just one  $\eta$ , then the mass matrices for the  $u$  and  $d$  quarks are related by

$$\mathcal{M}_d \langle \eta_1^0 \rangle^{-1} = \mathcal{M}_u \langle \eta_2^0 \rangle^{-1}, \quad (5)$$

which means that there can be no mixing among generations and the ratio  $m_u/m_d$  is the same for each generation. This is certainly not realistic and two  $\eta$ 's will be required.

$$\mathcal{M}_d = f \langle \eta_1^0 \rangle + f' \langle \eta_1'^0 \rangle, \quad (6)$$

$$\mathcal{M}_u = f \langle \eta_2^0 \rangle + f' \langle \eta_2'^0 \rangle. \quad (7)$$

As a result, the diagonalizations of  $\mathcal{M}_u$  and  $\mathcal{M}_d$  do not also diagonalize the respective Yukawa couplings, hence FCNC are unavoidable. To suppress these contributions to processes such as  $K^0 - \bar{K}^0$  mixing, the fine tuning of couplings is required if  $M_R \sim$  few TeV. In the nonsupersymmetric case,  $\eta'$  can be simply taken to be  $\sigma_2 \eta^* \sigma_2$ , but that will not alleviate the FCNC problem. Similarly, if the  $f'$  terms were radiative corrections from, say, soft supersymmetry breaking, FCNC would still be present.

### 3. Evolution of Gauge Couplings

Consider now the evolution of the gauge couplings to one-loop order.

$$\alpha_i^{-1}(M_U) = \alpha_i^{-1}(M_R) - \frac{b_i}{2\pi} \ln \frac{M_U}{M_R}, \quad (8)$$

where  $\alpha_i \equiv g_i^2/4\pi$  and  $b_i$  are constants determined by the particle content contributing to  $\alpha_i$ . Using the standard model to evolve  $\alpha_i$  from their experimentally determined

values at  $M_Z$  to  $M_R \sim \text{few TeV}$  and requiring that they converge to a single value at around  $10^{16}$  GeV, the constraints

$$b_2 - b_3 \sim 4, \quad b_1 - b_2 \sim 14, \quad (9)$$

are obtained. It is easily seen that these constraints are not satisfied by the conventional left-right gauge model with or without supersymmetry. Note that  $b_2 - b_3 = 4$  in the MSSM, corresponding to two  $\text{SU}(2)_L$  doublets, whereas in the supersymmetric left-right model with two bidoublets (four  $\text{SU}(2)_L$  doublets),  $b_2 - b_3 = 5$ .

#### 4. First Example with Unification

Suppose the FCNC problem is disregarded, then the conventional left-right model with particle assignments given by Eqs. (1) and (2) can be made to have gauge-coupling unification if new particles are added at the TeV energy scale.<sup>2</sup> Supersymmetry is also assumed so that  $M_R$  and  $M_U$  can be separated naturally. Now

$$b_S = -9 + 2(3) + n_D = -1, \quad (10)$$

$$b_{LR} = -6 + 2(3) + n_{22} + n_H = 3, \quad (11)$$

$$(3/2)b_X = 2(3) + 3n_H + n_D + 3n_E = 17, \quad (12)$$

and the constraints of Eq. (9) are satisfied. The gauge couplings do meet at one point as shown in Fig. 1, based on a full two-loop numerical analysis.

In this model  $n_{22} = 2$  is the number of bidoublets,  $n_H = 1$  is the number of an anomaly-free set of Higgs doublets needed to break the  $\text{SU}(2)_R$  symmetry independent of  $\text{SU}(2)_L$ :

$$\Phi_L \sim (1, 2, 1, -1/2), \quad \Phi_R \sim (1, 1, 2, 1/2), \quad (13)$$

$$\Phi_L^c \sim (1, 2, 1, 1/2), \quad \Phi_R^c \sim (1, 1, 2, -1/2), \quad (14)$$

$n_D = 2$  is the number of exotic singlet quarks of charge  $-1/3$ :

$$D \sim (3, 1, 1, -1/3), \quad D^c \sim (\bar{3}, 1, 1, 1/3), \quad (15)$$

and  $n_E = 2$  is the number of exotic singlet leptons of charge  $-1$ :

$$E \sim (1, 1, 1, -1), \quad E^c \sim (1, 1, 1, 1). \quad (16)$$

Note that  $n_{22} = 2$  and  $n_H = 1$  are required for fermion masses and  $\text{SU}(2)_R$  breaking respectively. To obtain  $b_{LR} - b_S = 4$ ,  $n_D = 2$  is then assumed. At this stage,  $(3/2)b_X - b_{LR} = 8$ . To increase that to 14,  $n_E = 2$  is just right. This should not be considered fine tuning because the contribution of each new set of particles comes in large chunks, 3 in the case of the  $E$ 's for example; so if 6 did not happen to be the desired number, it would not have been possible to achieve unification with the addition of new particles this way.

## 5. Left-Right Model without FCNC

Consider the  $E_6$  superstring-inspired left-right model proposed some years ago.<sup>4,5</sup> In the fundamental **27** representation of  $E_6$ , there is an additional quark singlet of charge  $-1/3$ . An alternative to the conventional left-right assignment is then possible:

$$Q \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L \sim (3, 2, 1, 1/6), \quad d_L^c \sim (\bar{3}, 1, 1, 1/3), \quad (17)$$

$$Q^c \equiv \begin{pmatrix} h^c \\ u^c \end{pmatrix}_L \sim (\bar{3}, 1, 2, -1/6), \quad h_L^c \sim (3, 1, 1, -1/3), \quad (18)$$

where the switch  $h^c$  for  $d^c$  has been made. The doublets  $\Phi_{L,R}$  and the bidoublet  $\eta$  are also in the **27**. Hence the following terms are allowed:

$$QQ^c\eta = dh^c\eta_1^0 - uh^c\eta_1^- + uu^c\eta_2^0 - du^c\eta_2^+, \quad (19)$$

$$Qd^c\Phi_L = dd^c\phi_L^0 - ud^c\phi_L^-, \quad (20)$$

$$hQ^c\Phi_R = hh^c\phi_R^0 - hu^c\phi_R^+. \quad (21)$$

As a result,

$$\mathcal{M}_u \propto \langle \eta_2^0 \rangle, \quad \mathcal{M}_d \propto \langle \phi_L^0 \rangle, \quad \mathcal{M}_h \propto \langle \phi_R^0 \rangle. \quad (22)$$

Since each quark type has its own source of mass generation, FCNC are now guaranteed to be absent at tree level. This is the only example of a left-right model without FCNC.

## 6. Extended Definition of Lepton Number

Since the  $(1, 2, 1, -1/2)$  component of the **27** is now identified as the Higgs superfield  $\Phi_L$ , where are the leptons of this model? One lepton doublet is in fact contained in the bidoublet, *i.e.*  $(\nu, l)_L$  should be identified with the spinor components of  $(\eta_1^0, \eta_1^-)$ , and one lepton singlet  $l_L^c$  with that of  $\phi_R^+$ . Since

$$\Phi_L\Phi_R\eta = \phi_L^-\phi_R^+\eta_1^0 - \phi_L^0\phi_R^+\eta_1^- + \phi_L^0\phi_R^0\eta_2^0 - \phi_L^-\phi_R^0\eta_2^+, \quad (23)$$

the lepton  $l$  gets a mass from  $\langle \phi_L^0 \rangle$ . Furthermore, from Eq. (19), it is seen that the exotic quark  $h$  must have lepton number  $L = 1$  and since  $u^c$  and  $h^c$  are linked by  $SU(2)_R$ , the  $W_R^-$  gauge boson must also have  $L = 1$ . This extended definition of lepton number is consistent with all the interactions of this model and is conserved.

The production of  $W_R$  in this model<sup>6,7</sup> is very different from that of the conventional left-right model. Because of lepton-number conservation, the best scenario is to have  $u + g \rightarrow h + W_R^+$ , where  $g$  is a gluon. The decay of  $W_R$  must end up with a lepton as well as a particle with odd  $R$  parity. Note also that  $W_L - W_R$  mixing is strictly forbidden and  $W_R$  does not contribute to  $\Delta m_K$  or  $\mu$  decay.

Since the absence of FCNC allows only one bidoublet, only one lepton generation is accounted for in the above. Let it be the  $\tau$  lepton. The  $e$  and  $\mu$  generations are

then accommodated in the  $\Phi_{L,R}$  components of the other two **27**'s, but they must not couple to  $Qd^c$  or  $hQ^c$ . This can be accomplished by extending the discrete symmetry necessary for maintaining the conservation of lepton number as defined above.<sup>3</sup>

## 7. Precision Measurements at the Z

Because of the Higgs structure of this model, there is in general some  $Z-Z'$  mixing which depends on the ratio of the  $W_L$  to  $W_R$  masses. Let  $\langle \eta_2^0 \rangle = v$ ,  $\langle \phi_{L,R}^0 \rangle = v_{L,R}$ ,  $r = v^2/(v^2 + v_L^2)$ ,  $x = \sin^2 \theta_W$ , then

$$M_{W_{L,R}}^2 = \frac{1}{2}g^2(v^2 + v_{L,R}^2), \quad (24)$$

and

$$M_Z^2 \simeq \frac{M_{W_L}^2}{1-x} \left[ 1 - \left( r - \frac{x}{1-x} \right)^2 \xi \right], \quad M_{Z'}^2 \simeq \frac{1-x}{1-2x} M_{W_R}^2, \quad (25)$$

where  $\xi = M_{W_L}^2/M_{W_R}^2$ . Deviations from the standard model can now be expressed in terms of the three oblique parameters  $\epsilon_{1,2,3}$  or  $S, T, U$ . Using the precision experimental inputs  $\alpha, G_F, M_Z$ , and the  $Z \rightarrow e^-e^+, \mu^-\mu^+$  (but not  $\tau^-\tau^+$ ) rates and forward-backward asymmetries, they are given by

$$\epsilon_1 = \alpha T = - \left( \frac{2-3x}{1-x} - r \right) \left( r - \frac{x}{1-x} \right) \xi, \quad (26)$$

$$\epsilon_2 = -\frac{\alpha U}{4x} = - \left( r - \frac{x}{1-x} \right) \xi, \quad (27)$$

$$\epsilon_3 = \frac{\alpha S}{4x} = - \left( \frac{1-2x}{2x} \right) \left( r - \frac{x}{1-x} \right) \xi. \quad (28)$$

Note that the ratio  $S/T$  must be positive and of order unity here. Experimentally,  $S, T, U$  are all consistent with being zero within about  $1\sigma$ , but the central  $S$  and  $T$  values are  $-0.42$  and  $-0.35$  respectively.<sup>8</sup> These imply that  $r \sim 0.8$  and  $\xi \sim 6 \times 10^{-3}$ , hence the  $W_R$  mass should be about 1 TeV which is exactly consistent with this model's assumed  $SU(2)_R$  breaking scale.

In this model, the  $\tau$  generation transforms differently under  $SU(2)_R$ , hence there is a predicted difference in the  $\rho_l$  and  $\sin^2 \theta_l$  parameters governing  $Z \rightarrow l^-l^+$  decay. Specifically,

$$\rho_\tau - \rho_{e,\mu} = 2 \left( r - \frac{x}{1-x} \right) \xi \sim 6 \times 10^{-3}, \quad (29)$$

compared with the experimental value of  $0.0064 \pm 0.0048$ , and

$$\sin^2 \theta_\tau - \sin^2 \theta_{e,\mu} = -x \left( r - \frac{x}{1-x} \right) \xi \sim -7 \times 10^{-4}, \quad (30)$$

compared with the experimental value of  $-0.0043 \pm 0.0022$ . The standard model's prediction for either quantity is of course zero.

## 8. Second Example with Unification

Fig. 2 shows the two-loop evolution of gauge couplings corresponding to the following situation. Let the particle content of the proposed left-right model be restricted to only components of the **27** and **27\*** representations of  $E_6$ , then unification is achieved with<sup>3</sup>

$$b_S = -9 + 2(3) + n_h = 0, \quad (31)$$

$$b_{LR} = -6 + 2(3) + n_{22} + n_\phi = 4, \quad (32)$$

$$(3/2)b_X = 2(3) + n_h + 3n_\phi = 18, \quad (33)$$

where  $n_h = 3$  and  $n_{22} = 1$  are required as already discussed, and  $n_\phi = 3$  is the number of extra sets of  $\Phi_L + \Phi_R + \Phi_L^c + \Phi_R^c$ . Note that at least one such set is needed for  $SU(2)_R$  breaking and that the two constraints of Eq. (9) are simultaneously satisfied with the one choice of  $n_\phi = 3$ .

To complete the model, six singlets  $N \sim (1, 1, 1, 0)$  are also assumed. At the unification scale  $M_U$ , there are presumably six **27**'s and three **27\***'s of  $E_6$ , which is then broken down to supersymmetric  $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)$  supplemented by a discrete  $Z_4 \times Z_2$  symmetry<sup>3</sup>. Of the three **27**'s and three **27\***'s, only the combinations  $\Phi_L + \Phi_R + \Phi_L^c + \Phi_R^c$  survive. Of the other three **27**'s, only two bidoublets do not survive. At  $M_R \sim \text{few TeV}$ ,  $\Phi_R$  and  $\Phi_R^c$  break  $SU(2)_R \times U(1)$  down to  $U(1)_Y$ . Supersymmetry is also broken softly at  $M_R$ . The surviving model at the electroweak energy scale is the standard model with two Higgs doublets but not those of the MSSM, as already explained in my first talk<sup>9</sup> at this meeting.

## 9. Lepton Masses

The  $\tau$  gets its mass from the  $\Phi_L \Phi_R \eta$  term, but there can be no such term for the  $e$  and  $\mu$ . Hence the latter two are massless at tree level. However, the soft supersymmetry-breaking term  $\Phi_L \Phi_R \tilde{\eta}$  (where  $\tilde{\eta} = \sigma_2 \eta^* \sigma_2$  and all three fields are scalars) is allowed, hence  $m_e$  and  $m_\mu$  are generated radiatively from the mass of the  $U(1)$  gauge fermion.<sup>10</sup> The neutrinos obtain small seesaw masses from their couplings with the three  $N_L$ 's which are assumed to have large Majorana masses. The  $\nu_\tau N_L$  mass comes from the  $\eta \eta N_L$  term, and the  $\nu_e N_L, \nu_\mu N_L$  masses come from the  $\Phi_L \Phi_L^c N_L$  terms.

## 10. Conclusion

New physics in the framework of left-right gauge symmetry is possible at the TeV energy scale even if grand unification is required. Two examples have been given, the second of which is particularly attractive: it is free of FCNC at tree level and has negative contributions to the oblique parameters  $S$  and  $T$  consistent with present experimental central values.

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